Problem Chosen	2022	Team Control Number
A	**MCM/ICM Summary Sheet**	**2220079**

Exhausted or Easy?

Summary

This paper focuses on the relationship between power and rider position in road cycling. A mathematical model that can be applied to any type of rider is developed through kinetic and biological principles. The limitations of the energy consumed by the rider and the limitations of the power curve are also considered. Finally, the model is extended to team competitions, and suggestions are provided for Directeur Sportif.

For task 1, we make the definition of power curve. We combine the meaning of the maximum output power p_max and CP during the rider's movement to derive the analytic formula of the function satisfied by the power profile, a hyperbola after translation, then, We fit the hyperbola using data from different riders, and build the power profile function expressions.

For task 2, we consider the limits for the rider with the knowledge of exercise physiology and mathematics by dividing it into two part and transfering them into mathematical constraints, then, we derive the equation relationship between the power and speed of the rider's movement, after that we consider the limitations of the sharp turns. Finally, by combining the above constraints and limitations, we derive a nonlinear planning model with the objective of minimizing the spending time and the configuration of the rated power of different sections as the decision variables, and finally, the planning model is solved by using the algorithm of particle swarm model.

For task 3, we consider taking the wind speed into account and modifying the dynamics model to obtain model 2-1. Then, we bring the wind speed between [-3m/s,3m/s] into the model for solving. The result shows that the completion time planned by the model fluctuates within 10% from the original time , indicating that the model is stable for different wind speed models.

For task 4, we do sensitivity analysis for different road sections and different power deviation degrees respectively and analyze the sensitivity of the model by the variation of distance as a function of time. The model is modified and solved by the deviation ratio λ of power. After that, We offset the power by a certain range and send it into the model the time results. It is obtained that a better distribution of the overall power is achieved when the rider's power deviation is ≤ 20 This shows the better stability of our model.

For task 5, we developed Model 3 to determine the best team collaboration model. We determined that the team was in a single line formation and designed the team's plan of the strategy for each rider. The overall team wind resistance was calculated to be 29.17%. The model was modified considering that the power curve would be exceed when riders took turns blocking the wind. Finally, the road section data was brought into the model to get the optimal power distribution for the team and the fluctuation of rider power.

Keywords: road bicycle, power curve, nonlinear programming model, dynamics analysis, particle swarm simulation algorithm

Contents

1 Introduction

Whether it is a standard time trial, a team time trial, or an individual time trial, the ability to accurately grasp the physical characteristics of a rider to rationalize his physical abilities and complete a specific course in the shortest possible time is a far-reaching and significant issue. The introduction of the power profile is to describe the difference between various types of riders, to accurately grasp the physical characteristics of riders.

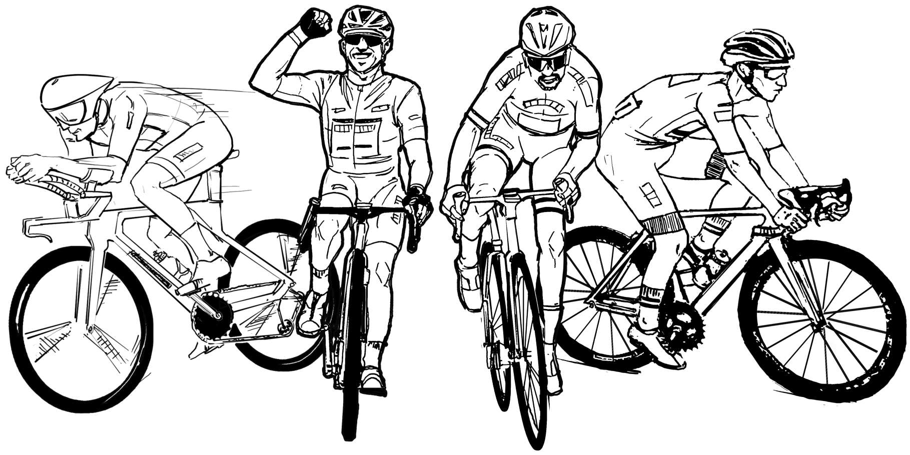

Figure 1: Different Categories of Cyclists

We built a model to precisely establish the problem of power distribution to the rider in different positions. In order that we could better distribute his physical power in all phases of the course, after which the wind speed and the team time trial were also taken into account. Finally, we analyzed the sensitivity of the model and reported our model to Directeur Sportif in order to achieve an improvement in the rider's performance.

1.1 Restatement of the Problem

Our ultimate goal is to determine the distribution of physical energy on different courses for different categories of cyclists(time trial specialist, a climber, a sprinter, a rouleur, or a puncher) based on their power profile, considering the total energy consumption of the rider and the cumulative effect of breaking the power profile, taking into account the composition of the course (e.g. road slope, length, curves) and the weather conditions.

- For more than two different categories and different genders of riders to define their power profile, the above categories must include time trial specialists.
- For different tracks, the model is used to determine the power distribution of the rider at different positions on the track, with the above track containing at least:
  - 2021 Olympic Time Trial course in Tokyo, Japan,
  - 2021 UCI World Championship time trial course in Flanders, Belgium,
  - At least one course of your design that includes at least four sharp turns and at least one nontrivial road grade. The end of the course should be near its start point.

- How the above model will be changed considering the influence of weather conditions, especially wind direction and wind speed and discuss the sensitivity of the model to changes in wind direction and wind speed.
- Consider a real-world situation where the rider has difficulty following the above power profile to the letter and will have some offset. Determine the impact of your model on the sensitivity of the power profile offset. The above analysis gives the rider and Directeur Sportif some understanding of the expected degree of change in key positions.
- Expand your model to include a team time trial with 6 people, based on the individual race above, where the teams time is determined when the fourth rider crosses the finish line.
- Last but not least, give a two-page riders race guidance for a Directeur Sportif of a team.

2 Prepartion of Models

2.1 Data Acquisition and Processing

This question does not provide us with the data directly, so we need to consider what data to collect in the model construction. By analysis of the problem, the data we need to collect are the rules of road cycling, the rider's indicators, the rider's past race results, the rider's power, etc. Since there are too many different types of data to list, visualizing the data is a good way to visualize the data.

2.1.1 Data Collection

By checking the official UCI website, we got a lot of information about the road cycling rules and the results of the riders' races. Also obtained was the road information for the races requested in the title was also obtained. Additional data sources are shown in Table 1

Table 1: Data and dataset websites

Category	Resources
Articles	`https://scholar.google.com/`
Altitude	`https://search.earthdata.nasa.gov/search`
Maps	`https://www.komoot.com/discover`
Road	`https://earth.google.com`
Riders Information	`https://github.com/GoldenCheetah/OpenData`

2.1.2 Data Screening

This map shows our design of the race route, the central route, with the elevation of the different areas at the bottom. We can also use this generating a gpx file to get more information through MatLab and google earth.

Here is a map visualization of the Belgian championship race processed in Matlab

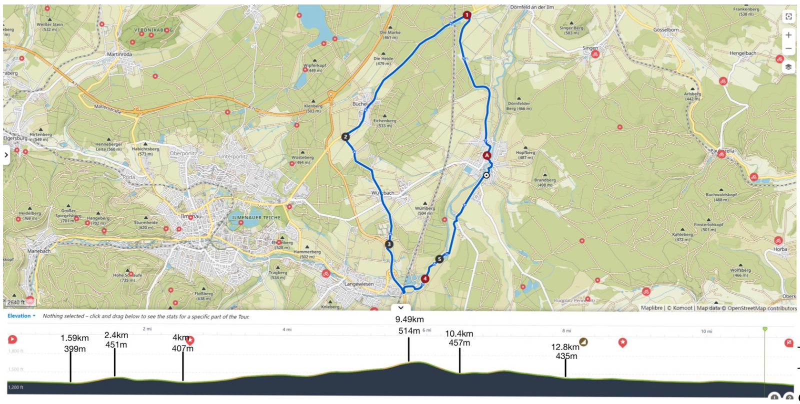

Figure 2: Design our own Track

Figure 3: Process Belgium Track

2.2 Assumptions and Justifications

Considering that the actual problem always contains many complex factors, we first make reasonable assumptions to simplify the model and justify the reasonableness of the model.

- **Hypothesis 1: All types of tracks are divided into flat, slopes, and curves while ignoring other elements.**
  Explanation: The requirements for self-designed tracks in this topic are: includes at least four sharp turns and at least one nontrivial road grade.Therefore, for the sake of simplicity, we only consider road compositions that include flat land, slopes, and curves.

- **Assumption 2: All slope paths are considered as ideal slopes considering only slope and friction.**
  Explanation: The actual moving process is very complex, and it is not feasible to build a physical model based entirely on reality, so we only select the most important elements for analysis.

- **Assumption 3: Consider the road surface as an ideal plane and ignore the error due to the unevenness of the actual road surface.**
  Explanation: Since the actual data does not provide a good measure of the energy consumption generated by road surface undulations, we only analyze the most critical factors to simplify the model.

- **Hypothesis 4: Consider only the translational effect produced by the wind and ignore the rotational effect it produces.**
  Explanation: The cause of the natural wind is the flow of gases. The effect of rotation has a very small effect on the rider compared to the effect of flat movement. Therefore, we reduce the effect of wind to a vector with direction.

- **Hypothesis 5: The width of the road can be ignored in considering the rider on the road.**
  Explanation: This is because the width of the road is very small compared to the length of the road or the radius of curvature corresponding to the turnaround. Neglecting their widths does not have a significant impact on the model results.

  In addition, some assumptions are made to simplify the analysis of different parts. These assumptions are explained in the appropriate section.

2.3 Notations

Table 2: Symbols and descrpitions

Variable Name	Description	Data Type
Ps	ratio of power delivered by the rider to the bike's pedals to the rider's mass	w/kg
$td(Ps)$	the maximum duration that the rider can deliver with power Ps	s
W_{max}	the maximum amount of energy the rider can transfer to the bike's pedals	J
S	distance rider rides over	J
m	mass of rider	kg
$F(t)$	the power generated by the rider to the bicycle	N

3 Models and Solutions

3.1 Model 1: Power profile model for different categories of cyclists

3.1.1 Modeling

This question asks us to define the power profile of different kinds of riders, taking into account the influence of gender. First, we should define the meaning of the power profile. Then, we construct a functional expression for the power profile based on the power (hereinafter referred to as "power" for simplicity) transferred to the pedals of the bike by different types of riders and their duration. **Definition: The power profile** can assess more physical capacities from the relationship between the power output of the riders (Ratio to body m) and the maximum time(between 1 s and 4 h) that riders can maintain.[1].

From the above definition we understand that the power profile is a graph of the ratio Ps to the rider's power-weight as a function of its maximum time of continuous movement at this power $td(Ps)$.

According to the relationship between work and energy, if the variation of rider power with time is $P(t)$,we can obtain:

$$W(t) = \int P(t)dt$$

And by the definition of td and Ps, we know that when the rider transfers energy to the pedals with the power of Ps, the rider is able to keep riding continuously for a maximum of td time, so $P(t) = Ps \times m$,the above equation can be reduced to

$$W_{max} = td(Ps) \times Ps \times m \tag{1}$$

Since the rider's weight does not change approximately during the ride, the above equation inspires us: the rider's power P and its maximum time of continuous motion at this power $td(Ps)$ should approximately satisfy a form similar to the inverse proportional function.

Considering the practical situation, on the one hand, since cyclists can perform long rides of up to 4h or more, this implies the existence of an infinitely continuous critical value of the rider's power, which we call: "Critical power" (CP).

It should represent the horizontal asymptote of the power profile [2]. However, since one cannot ride indefinitely, such a CP does not exist. In general, we generally choose the power on the power profile corresponding to 40 minutes as the value of Critical power)[4].

On the other hand, there is an upper limit for the rider's sprint power (e.g. $td(Ps) < 1s$) and thus it cannot increase without limit, thus there is an upper limit of power per rider Ps_{max}[3].

After considering the actual situation, we can make the following corrections to the form of the formula 1:

$$W = (Ps - CP)(td(Ps) - a)$$

where,W is the constant that needs to be determined by fitting.

When $td(Ps) = 0$, $Ps = Ps_{max}$. Thus, we can obtain the power profile model modified for the real situation as follows:

$$td(Ps) = W\left(\frac{1}{Ps - CP} - \frac{1}{Ps_{max} - CP}\right) \tag{2}$$

After that, we apply the formula 2 to fit the data for different sexes and different categories of riders with the objective of minimizing the LSE, and obtain the following results:

Table 3: Power profile for different categories of riders

Category	gender	power profile expressions	R^2
All-Rounder	Male	$td(Ps) = 288\left(\frac{1}{Ps-3.98} - \frac{1}{9.77-3.98}\right)$	92.94%
Sprinter	Male	$td(Ps) = 374.5\left(\frac{1}{Ps-3.32} - \frac{1}{11.08-3.32}\right)$	93.83%
Time Trialist	Male	$td(Ps) = 273.1\left(\frac{1}{Ps-4.91} - \frac{1}{8.83-4.91}\right)$	91.65%
Pursuiter	Male	$td(Ps) = 409.5\left(\frac{1}{Ps-4.23} - \frac{1}{7.51-4.23}\right)$	90.72%
All-Rounder	Female	$td(Ps) = 194\left(\frac{1}{Ps-3.36} - \frac{1}{8.91-3.36}\right)$	80.83%
Sprinter	Female	$td(Ps) = 294.8\left(\frac{1}{Ps-3.00} - \frac{1}{10.84-3.00}\right)$	95.63%
Time Trialist	Female	$td(Ps) = 243.1\left(\frac{1}{Ps-4.45} - \frac{1}{8.25-4.45}\right)$	95.81%
Pursuiter	Female	$td(Ps) = 331.8\left(\frac{1}{Ps-4.03} - \frac{1}{7.28-4.23}\right)$	94.93%

Afterwards, we will visualize the results described above:

3.1.2 Presentation of the Conclusion

First, since sprinters have a high power output in a short period, their power profile is more typical, so we first show the power profile of male and female sprinters, and then we show the power profile of male and female time trial specialists as required by the question.

Last but not least, to visualize the differences in power curves between different kinds of riders, we put the power curves of different kinds of male riders in the same graph to visually compare the differences.

It is easy to see from the figure that the sprinters have a high Ps_{max}. Because his power curve decays faster with duration, he has a lower CP. Pursuiter can maintain more power for a shorter period ($td < 500s$), however, his Ps_{max} differs

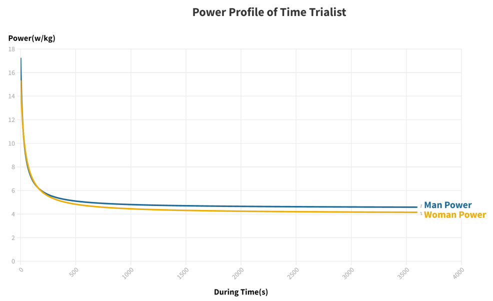

Figure 4: Power Profile of Trialist

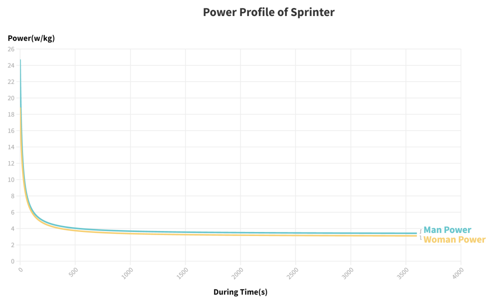

Figure 5: Power Profile of Sprinter

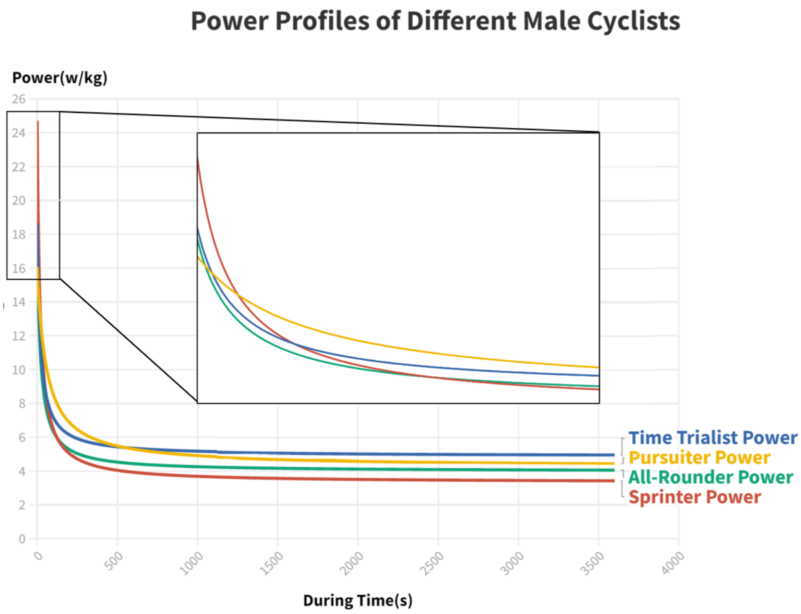

Figure 6: Power Profiles of Different Male Cyclists

from sprinters, while the time trial specialist is the rider with the best speed retention, he has the maximum CP, which means he can ride longer with more, and this is the key to his frequent wins. Therefore, the above power curve matches the actual situation.

3.2 Model 2: Kinetic model based on road conditions and athlete power curves

The goal of this question requires us to base on the actual situation of the track, for different track types, and to make a reasonable allocation of power to the rider according to his position, taking into account the limitations of the rider's power curve, in order to achieve the goal of the rider being able to pass the specified section in less time.

In the process of power allocation, the above track types are specified as follows

- flat and straight road
- Sloping roads with little slope
- Curved road

Also, consider the following restrictions.

- energy consumption
- the effect of previous sprint process
- the effect of briefly breaking the power curve

3.2.1 model of fatigue accumulation and energy limitation

fatigue accumulation

The causes of rider fatigue during riding can be divided into two types, one is the fatigue accumulated by the rider performing a short sprint, and the other is the energy consumed by the rider breaking the power curve.

First, consider the fatigue accumulated by the rider performing short-distance sprints. According to the knowledge of sports exercise physiology [5], the reason why the human body shows a short duration at higher power, long duration at higher power, and a power curve with a hyperbolic-like shape is determined by the different energy supply characteristics of the human body at different powers.

Table 4: different energy supply stages of human body

Type	duration	characteristics	recovery time
ATP+CP System [1]	$0-20s$	short duration, fast acting, for explosive items	4mins[5]
Glycolysis System	$6s-3mins$	performs anaerobic respiration, produces lactic acid	
Aerobic system	$\leq 2mins$	longer duration, slower action, for ongoing projects	none

Table 4 visualizes that: the ATP+CP and Glycolysis phases last no more than 3mins at most, which explains that when $td \leq 3mins$, all the exercises they do are anaerobic with high explosive capacity and take lactic acid as a metabolite, while the accumulation of lactic acid makes the muscles tired. In general, after 3mins of anaerobic exercise, the accumulation of lactic acid will reduce the power output of the muscle by 2%[6]. And after 3mins of rest can restore the energy supply capacity of ATP+CP with Glycolysis system. [5]

Therefore, we can represent the above exercise physiology of the rider's sprint process in mathematical language and model its proper description as follows: when the rider rides at a power of $td \leq 3mins$, entering the sprint phase, the muscles complete a large amount of anaerobic respiration and accumulate lactic acid, thus causing a 95% shift in the power curve, which changes to

$$td_n(Ps) = (0.95)^n \times W\left(\frac{1}{Ps - CP} - \frac{1}{Ps_{max} - CP}\right)$$

Where: n is the number of times the rider enters the above sprint phase.

And when the rider persists in td s sprint at this power, the power will quickly fall back to the critical power position and wait for 3mins before entering the sprint phase again because his body can no longer support him to finish supplying energy.

After **, consider the energy consumed by the rider to break through the power curve.** The rider can briefly break his power curve limit, and the extra power thus brought is compensated by the ATP+CP System [5], so it also accumulates lactic acid and reduces the energy output by $2\%W_{max}$ and lasts $td \le 20s$, before breaking the energy curve again after a 3mins rest.

The descriptive model of the above rider breakthrough power curve is as follows: when the rider rides at Ps power, the maximum breakthrough time is $td(Ps) + 20s$, the muscles complete high-intensity anaerobic respiration and consume a lot of energy, thus, each breakthrough of the power curve limit will make the rider reduce $2\%W_{max}$ energy reserve and take 3mins to recover, i.e.

$$W_{nmax} = (1 - 2n\%)W_{max} \tag{3}$$

where n is the number of times the power curve is breached.

energy limitation

For the energy supply limit, since Critical Power is the maximum power corresponding to the rider being able to ride 40mins, we can then obtain the approximate energy reserve of the rider as

$$W_{max} = 2400 \times CP \times m \tag{4}$$

The above is the mathematical model of the above three limits.

3.2.2 dynamic analysis model of bicycle driving process

The bicycle travel process is driven by the rider through the pedals to drive the crank and sprocket, and then through the chain to transfer the rotation to the rear wheel, and then produce the process of movement, and at the same time, to overcome the air resistance, up and downhill generated for the decomposition of gravity, so we try to analyze the energy changes in the process of bicycle movement in order to establish a proper mathematical model.

As shown in Figure 14, in the process of driving, the rider and the overall of the bicycle will be subject to gravity, ground support, air resistance and friction, wheel rotation and lower limb pedaling action will produce the Magnus effect. [8]

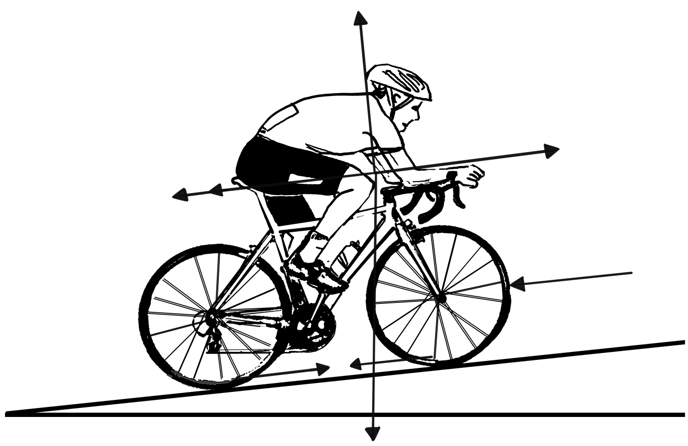

Figure 7: Force analysis of a bicycle at steady state

Let the power of the rider to the bicycle pedals be $p(t)$ for a person at t moment, the velocity of the bicycle movement be $v(t)$, and the efficiency of the sprocket-chain system in transferring energy be η, then the power $F(t)$ generated by the rider to the bicycle is

$$F(t) = \eta \, \frac{p(t)}{v(t)} \tag{5}$$

For the uphill phase, note that the inclination of the slope is θ, then gravity can be decomposed into a force F_p down along the slope and a force F_v perpendicular to the slope, and F_v is balanced with the support force F_n of the slope on the rider, thus

$$\begin{aligned} F_N &= F_v = mgcos\theta \\ F_p &= mgsin\theta \end{aligned} \tag{6}$$

The rolling friction coefficient between the wheel tire and the slope is C_r. Its magnitude is determined by the road surface and the material of the tire. Then the frictional force F_f of the slope on the rider is

$$F_f = C_r F_N = C_r mgcos\theta \tag{7}$$

where C_r is the rolling friction coefficient between the wheel tires and the ground, and this coefficient depends on the road surface and the material of the tires.

When the human-vehicle system advances, the air in front is compressed to produce pressure, and the friction between the surfaces on both sides and the air produces friction, all these forces are opposite to the direction of motion, called the air resistance. The action part of the air resistance is approximately coincident with the center of mass, and its expression is

$$F_w = \frac{1}{2} C_d A \rho V^2 \tag{8}$$

where C_d is the air drag coefficient, A is the maximum cross-sectional area of the windward side, ρ is the air density at the altitude where it is located, and V is the relative velocity of the vehicle body with respect to the air flow.

As the wheel is constantly rotating, it generates drag in the air. The resistance generated by rotation depends mainly on the wheel size and wheel shape and has little to do with the wheel speed. The air resistance to the rear wheel is reduced by 25% due to the action of the human body and the bicycle vertical beam. For a typical road bike with equal front and rear wheel size, the formula for rotational resistance is

$$F_{\mathrm{d}} = F_{\mathrm{fd}} + F_{\mathrm{rd}} = \frac{1}{2} C_{\mathrm{w}} \rho V^2 \pi r^2 + \frac{3}{4} \times \frac{1}{2} C_{\mathrm{w}} \rho V^2 \pi r^2 = \frac{7}{8} C_{\mathrm{w}} \rho V^2 \pi r^2 \tag{9}$$

where C_w is the air resistance coefficient of the bicycle wheel and r is the wheel radius.

According to Newton's second law, the kinetic equation of the bicycle in motion can be expressed as

$$F - F_g' - F_f - F_w - F_d = M \cdot a \tag{10}$$

where F is the forward force generated by the athlete's pedal stroke. Since the pedal force is not a constant force, the force F can also be expressed as the ratio of power to velocity, i.e., $F = \eta \, \frac{p(t)}{v(t)}$. In the process of pedal-driven chain rotation and chain-driven wheel rotation, there will be part of energy loss, and in general, the chain transfer loss of road bicycle is $\eta = 98.5\%$.

According to the above force analysis, the kinetic equation satisfied by the road bicycle in the forward process can be written as:

$$\eta\frac{p(t)}{v(t)} - mgsin\theta - C_r mgcos\theta - \frac{1}{2}C_d A\rho v^2(t) - \frac{7}{8}C_w \pi r^2 v^2(t) = \left(m + \frac{I_f + I_r}{r^2}\right)\frac{dv(t)}{dt} \quad (11)$$

where I_r and I_f denote the rotational inertia of the front and rear wheels, respectively.

The model parameters are shown in the following table

Table 5: The List of the Initial Parameter of Model

Indicator category	value
Environmental indicators	$\mathrm{T} = 20°\mathrm{C}, \mathrm{P} = 1\ \mathrm{atm}, \rho = 1.266\ \mathrm{kg/m^2}$
Body index	$\mathrm{M} = 80\ \mathrm{kg}, \mathrm{H} = 180\ \mathrm{cm}, \mathrm{Cd} = 0.5,\ \mathrm{A} = 0.5\ \mathrm{m^2}$
Body index	$\mathrm{r} = 0.35\ \mathrm{m}, \mathrm{If} = \mathrm{Ir} = 0.08\ \mathrm{kg \cdot m^2}, \mathrm{Cw} = 0.0397, \mathrm{Cr} = 0.004$

3.2.3 model of curves in the track

Curves on the track are divided into sharp and gentle curves according to their radius of curvature. Generally speaking, for gentle curves, their paths are longer, but their radius of curvature is small, resulting in a smaller centripetal force required and a larger centripetal force F_n required for turning, thus requiring a smaller frictional force F_f, and therefore not much energy consumption for bicycle driving, for sharp curves, although their radius of curvature is larger and turning For sharp bends, although their radius of curvature is larger and the centripetal force F_n needed to turn is large, resulting in a larger fraction of the friction force F_f consumed, however, their paths are generally shorter, so again there is no large energy consumption for the bicycle [13].

However, from the perspective of mechanical equilibrium, sharp bends still have an impact on our movement. This is due to its small radius of curvature, which is given by

$$F_n = m\frac{v^2}{r}$$

It can be seen that the centripetal force F_n required for turning is large, so the problem of mechanical equilibrium needs to be considered. At the same time, because the centripetal force F_n is not in the same plane with the support force, gravity, and friction force, it leads to the problem of moment balance.

In order to achieve moment balance, the rider uses body tilt in order to pass the curve smoothly.

From the knowledge of mechanics, note that θ is the angle with the ground when the vehicle is tilted,L is the position of the body's center of gravity, then from the moment balance it is obtained that

$$mgLcos\theta = m\frac{v^2}{r}Lsin\theta$$

Thus it is obtained that

$$v_{max} = \frac{tan\theta}{grL} \quad (12)$$

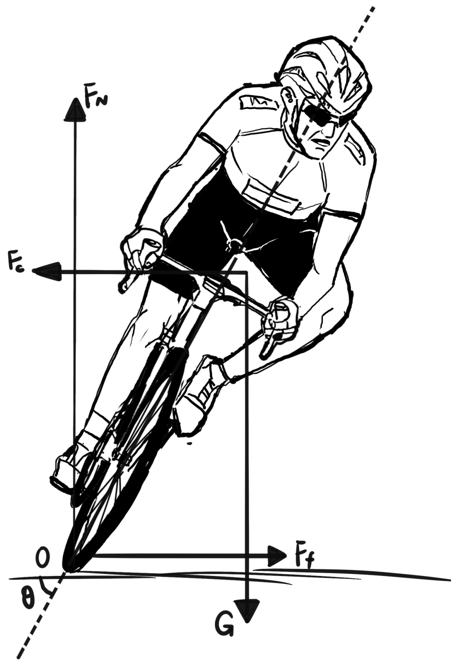

Figure 8: Analysis of the forces on the bicycle when tilted

In general, there is an upper limit to the θ inclination value of the human body, so the maximum velocity v_{max} of a section can be obtained by finding the minimum radius of curvature r corresponding to a sharp slope on a section

3.2.4 planning model with integrated constraints

Since the shape of the power profile has the following characteristics: the power has a faster decay with time, and the duration of a typical individual cycling time trial is above 20 minutes, the average power $\overline{Ps}$ chosen by the runner during the march should satisfy at least $td(\overline{Ps}) > 1200s$, and The shape of the power profile determines that the power value corresponding to $td(\overline{Ps}) > 1200s$ does not vary too much, and frequent changes in the current frequency are extremely physically demanding for the runner [9], therefore, the guideline for individual time trials is generally " maintain a steady power output for long periods"[7]. Therefore, we assume in advance that the rider's power remains constant under a certain road condition (e.g., in an uphill phase). Thus we first assume that the rider's power is maintained constant at $P = 500w$, and for a slope with an inclination angle of $2°$ and a flat surface, solve the differential equation shown in Eq. 11 for a series of given initial speeds, and plot the speed as a function of time, as shown in Fig. 10.

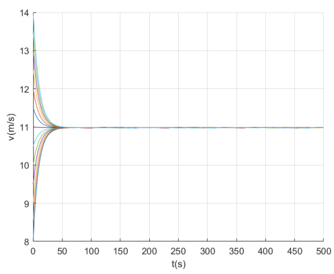

Figure 9: Uphill

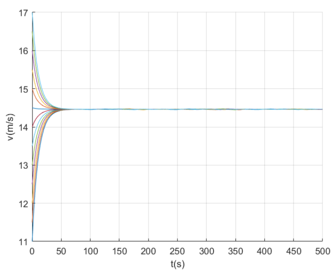

Figure 10: Flat Road

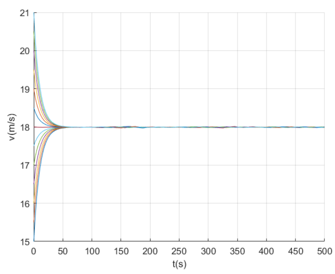

Figure 11: Downhill

We then selected several sets of slope and initial velocity data, solved them, and finally found an interesting conclusion: **When the power is constant, for a given road condition information, even if the rider's initial velocity is different, after a short period of time (about 60s), the rider will approximately maintain a constant velocity (noted as v_c below)**. Thus, due to the long duration of the individual time trial, we can ignore the time when the initial speed changes (60s) and approximate that when the power is constant, the speed also remains constant.

Thus, given a range of powers P for a particular road condition and calculating the speed v_c at which the rider can run at a uniform speed with this power and finding a good linear relationship between the two with little variation, we consider that P has the following linear relationship with v_c.

$$P = kv_c + b$$

After that, we fit different road conditions for 2021 Men's Olympic Time Trial course in Tokyo, Japan,2021 Men's UCI World Championship time trial course in Flanders, Belgium, Our course Using linear regression to fit P as a function of v_c, limited to space, only the fit for 2021 Men Olympic Time Trial course is shown here.

Table 6: 2021 Men's Olympic Time Trial Course Function

category	angle	function expression	R^2
ramp	$-2.0787°$	$P = 0.8233v_c + 5.2452$	88.03%
Slope	$-0.9896°$	$P = 0.6958v_c + 5.8686$	91.67%
Slope	$2.3436°$	$P = 0.9231v_c + 7.1163$	91.45%
Slope	$-1.9385°$	$P = 0.8023v_c + 5.3792$	87.01%
Sloping road	$2.5766°$	$P = 0.9842v_c + 8.8958$	86.62%
Flat road	$0.0125°$	$P = 0.5341v_c + 5.0595$	90.82%

Since we assume in advance that the rider is riding at rated power on a certain road condition, and by the above analysis, we will end up with a uniform motion in a short time regardless of the rider's initial speed, if we remember that we have a total of n different road sections, the length of the i section is S_i and the speed is noted as: v_{ci} Therefore, the total time used is

$$T = \sum_{i=1}^{n} \frac{S_i}{v_{ci}} \tag{13}$$

Considering the equation 3,i.e., the power consumption caused by the rider temporarily breaking the limits of the power curve, we can obtain the power consumption of the rider as

$$\sum_{i=1}^{n} p_i \frac{S_i}{v_{ci}} + \frac{1}{2}\left(sgn(\frac{S_i}{v_{ci}} - td_i) + 1\right) 2\% W_{max} \tag{14}$$

Consider the equation 12, remember to consider the maximum speed of the i kind of road section of the corner limit is v_{imax}, then of that the rider's speed can not lead to fly out of the sharp turn area, we can get its speed limit:

$$v_i \leq v_{imax}, \quad i = 1, \cdots, n \tag{15}$$

Considering the limits of the power curve, and writing td_i as the maximum duration of motion with power p_i, then considering the limits of the power curve (or temporarily breaking) the power curve, we get

$$\frac{S_i}{v_i} \leq td_i + 20 \quad i = 1, \cdots, n \tag{16}$$

Let p_i of each segment be related to v_{ci} as a function of

$$v_{ci} = k_i p_i + b_i$$

and bringing it with the equation 2 into the above equation 13-16, we can obtain the following comprehensive model.

$$\min \sum_{i=1}^{n} \frac{S_i}{k_i p_i + b_i}$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} p_i \frac{S_i}{k_i p_i + b_i} + \frac{1}{2}\left(sgn\left(\frac{S_i}{k_i p_i + b_i} - W_i\left(\frac{1}{P_i - CP_i} - \frac{1}{Pi_{max} - CP_i}\right)\right) + 1\right) 2\% W_{max} \leq W_{max} \\ k_i p_i + b_i \leq v_{imax} \quad i = 1, \cdots, n \\ \frac{S_i}{k_i p_i + b_i} \leq W_i\left(\frac{1}{P_i - CP_i} - \frac{1}{Pi_{max} - CP_i}\right) + 20 \quad i = 1, \cdots, n \end{cases} \tag{17}$$

The $sgn(x)$ in the above equation is a symbolic function.

The above nonlinear optimization model takes $(p_1, \cdots, p_n)$ as the optimization variables and solves the equation 17 to obtain the optimal power allocation at each section S_i using the particle swarm simulation algorithm(fig3.2.4).

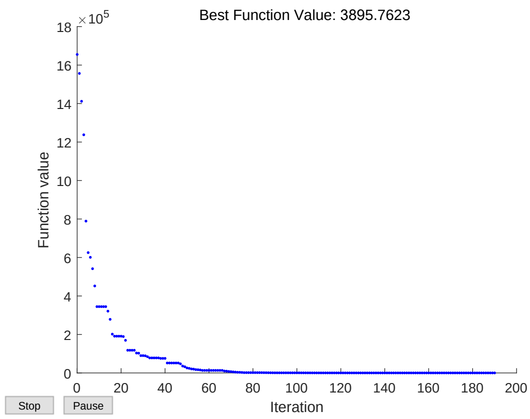

Figure 12: particle swarm simulation algorithm

In the following for 2021 Men's Olympic Time Trial course in Tokyo, Japan,2021 Men's UCI World Championship time trial course in Flanders, Belgium, Our course, we invoke the planning model shown in Eq. 17 to obtain the following power allocation with position

3.3 model2 of modification 1: dynamics model considering weather conditions

In this problem, we need to consider the effect of wind speed and wind direction on the rider. Since the equation 11 does not consider the wind, we need to make a correction to the model based on the equation 14, and note that $v_w(t)$ is the rider's head-on wind speed, then the modified model is as follows:

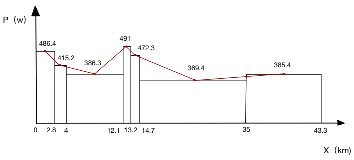

Figure 13: Belgium

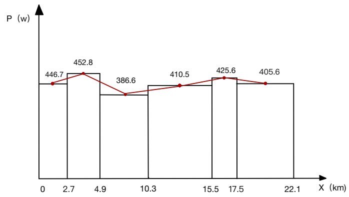

Figure 14: Tokyo

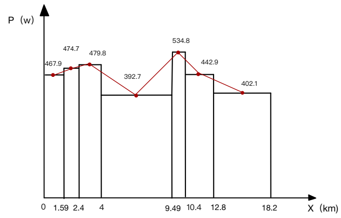

Figure 15: Own Track

$$\eta\frac{p(t)}{v(t)} - mgsin\theta - C_r mgcos\theta - \frac{1}{2}C_d A\rho\left(v(t)+v_w(t)\right)^2 - \frac{7}{8}C_w\pi r^2 v^2(t) = \left(m+\frac{I_f+I_r}{r^2}\right)\frac{dv(t)}{dt} \quad (18)$$

After that, we re-fit P as a function of v_c for $v_w = 1m/s, -1m/s, 3m/s, -3m/s$ winds and bring the equation 17 to optimize the solution, and finally obtain the following plot of the wind speed variation as a function of distance as a function of time.

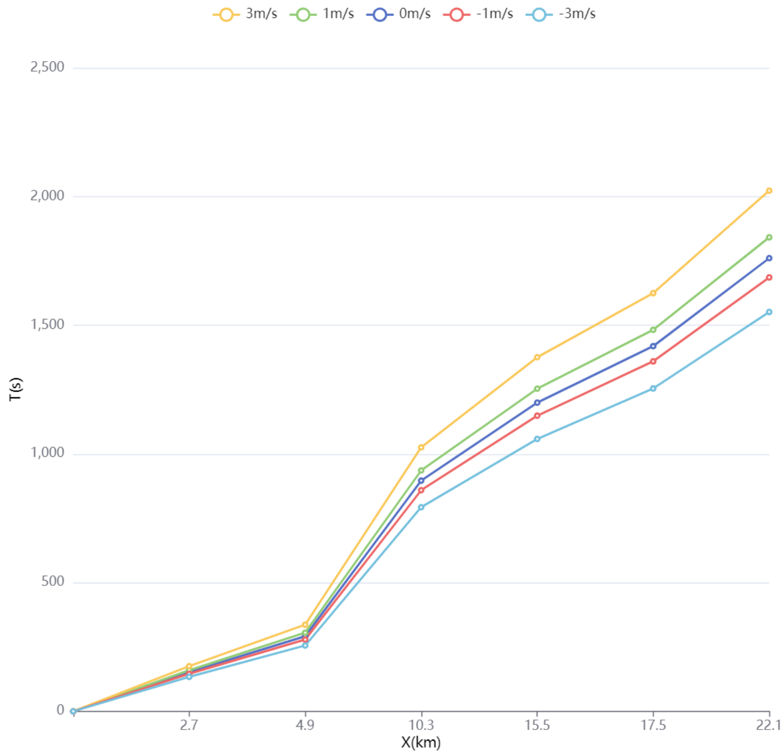

Figure 16: Sensitivity analysis of power regarding wind speed

As shown in the figure above, the integrated model is stable for different wind speeds. In the wind speed range of plus or minus 3m, the completion time obtained from the integrated model is within 10%, which proves that our model has strong stability.

However, in actual road racing, the natural wind is not parallel to the rider's movement direction, and usually, the natural wind can only maintain a constant wind direction and wind speed for a certain period of time. However, since the path of the track in this problem is a closed loop, the paths along the wind direction are also approximately equal, and since our model has strong stability with respect to the effects of wind, the rider's power distribution does not change significantly under the influence of natural wind along a particular direction.

3.4 Modification 2 of Model 2: Analysis of the sensitivity of the model to rider deviation from the target power

The objective of this problem is to determine the sensitivity of the rider to deviations from the target power distribution and to give the possible range of expected split times in the key parts of the course when the rider deviates from the target power distribution. In general, a rider can't distribute power exactly as desired, so we need to determine how the rider's actual power deviates from the desired power as a function of the time he takes to travel a certain distance.

In this problem, we take the 2021 Men's Olympic Time Trial course as an example, firstly, we divide the road into sections according to the road conditions, and then we study the effect of the rider's deviation from the ideal power on the rider's time at different sections. Finally, we also investigated the extent to which the rider's time changed under different degrees of deviation from the ideal power for a given section.

A better rider rarely or rarely deviates from the ideal power distribution, so we only consider the effect of the rider's deviation from the target power distribution in a given section (not the whole course).

Assuming that on the ith section, the rider's power distribution will have a corresponding deviation, the percentage of deviation is λ, i.e. $p_{inew} = (1-\lambda)\, p_i$ then the equation 17 will be modified as follows:

$$
\min \sum_{j=i+1}^{n} \frac{S_j}{k_j p_j + b_j} + \frac{S_i}{k_i p_{inew} + b_i}
$$

$$
\text{s.t.} \begin{cases}
\sum_{j=1,j\neq j}^{n} p_j \frac{S_j}{k_j p_j + b_j} + \frac{1}{2}\left(sgn\left(\frac{S_j}{k_j p_j + b_j} - W_j\left(\frac{1}{P_j - CP_j} - \frac{1}{Pj_{max} - CP_j} \right)\right) + 1\right) 2\% W_{max} + \\
p_{inew} \frac{S_i}{k_i p_{inew} + b_i} + \frac{1}{2}\left(sgn\left(\frac{S_i}{k_i p_{inew} + b_i} - W_i\left(\frac{1}{P_j - CP_j} - \frac{1}{Pj_{max} - CP_j} \right)\right) + 1\right) 2\% W_{max} \leq W_{max} \\
k_j p_j + b_j \leq v_{jmax} \quad j = 1, \cdots, i-1, i+1, \cdots, n \\
k_i p_i + b_i \leq v_{imax} \\
\frac{S_j}{k_j p_j + b_j} \leq W_j \left(\frac{1}{P_j - CP_j} - \frac{1}{Pj_{max} - CP_j} \right) + 20 \qquad j = 1, \cdots, i-1, i+1, \cdots, n \\
\frac{S_i}{k_i p_i + b_i} \leq W_i \left(\frac{1}{p_{inew} - Cp_{inew}} - \frac{1}{Pi_{max} - Cp_{inew}} \right) + 20 \quad i = 1, \cdots, n
\end{cases} \tag{19}
$$

Based on the modified formula, the particle swarm algorithm was applied to optimize the power distribution. After that, for the case where the rider produces $\lambda = 10\%, 20\%, -10\%$ deviations in different sections of the track, the formula 19 is brought into the calculation and finally determines the optimal power distribution after its deviation and calculates the range of split times given by its key parts in the track 7 is the range of split times given at different key parts for the first section when the power deviation is generated.

Table 7: The table of Split Times

Range of changes	9.7km	15.0km	22.1km	31.8km	37.1km	44.2km
0	$882s$	$1297s$	$1639s$	$2593s$	$2941s$	$3764s$
-10%	$942s$	$1414s$	$1659s$	$2683s$	$3051s$	$3853s$
10%	$957s$	$1454s$	$1712s$	$2789s$	$3124s$	$3900s$
20%	$980s$	$1500s$	$1850s$	$2850s$	$3330s$	$3953s$

From the individual Split Times in 7, it can be seen that when the rider's power generation changes$\leq 20\%$, the range of their split times 5% and can achieve a better distribution of the overall power, which shows that our model is more stable for riders deviating from the stability of the power distribution is better, and 1also provides a visual illustration of this result.

Subsequently, we also performed an analysis of the effect of the same power fluctuation on the completion time of different sections of the road, as shown in the following figure shown below

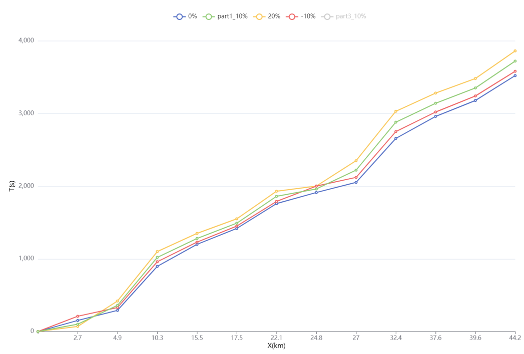

Figure 17: Sensitivity analysis of power variation in the same section

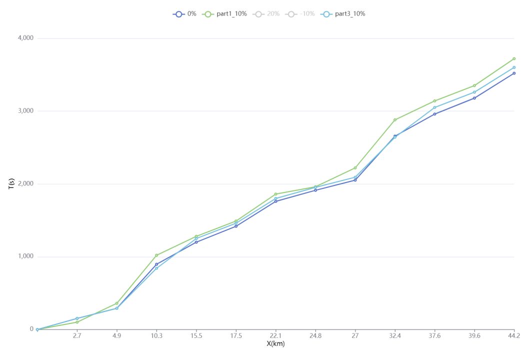

Figure 18: Sensitivity analysis of the same power variation in different road sections

The above figure shows the power distribution curve for the first and the third part with a 10% power offset at the same time. Therefore, no matter where the rider deviates from the ideal power, our integrated model is better. Therefore, regardless of the rider's deviation from the ideal power, our integrated model can adjust the subsequent power allocation to achieve the shortest time to finish the race with high stability within the total energy limit.

3.5 Model 3: Optimal Teamwork Model

This topic requires us to find an optimal power allocation for a 6-person system team time trial with the fourth person completion time as the performance criterion to minimize the overall team time.

For the team time trial, its team collaboration model generally can be divided into single-line type, two-line type and three-line type [10]. Specifically for the number of people and rules of the team time trial required by this topic, the current mainstream team collaboration mode is mainly single-line type, and the action strategy generally adopts the following rotation strategy[11] as figure 19

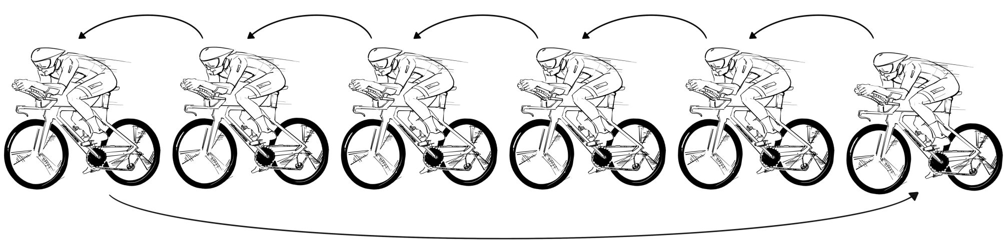

Figure 19: Team marching strategy

The specific process of the above rotation strategy is as follows: first, the 1st person is at the front of the team, blocking the wind resistance for the players behind him. Therefore, during this time, this player will use as much power as possible, so he will break through the power curve, and the breakthrough time is around 20 seconds according to the equation 3. And after that, he will go back to the last place of the team and a second teammate will take over to block the wind resistance.

Generally speaking, the rider in front of the team will bear 100% of the original wind resistance, the riders in second and third place in the team will bear 80% and 65% of the original wind resistance, and the riders in fourth to sixth place will bear 60% of the original wind resistance. Therefore, for the team as a whole, this rotation mode reduces the wind resistance of the team as a whole as follows

$$100\% - \frac{100\% + 80\% + 65\% + 60\% * 3}{6} = 29.17\%$$

Then, to simplify the model, we assume that all riders have the same physical fitness. Therefore, we can consider the cycling team as a whole. Although each rider will often change his power, the cycling team as a whole is still moving at rated power. Therefore, for equation 11, considering the reduction in wind resistance due to teamwork, we modify it as follows

$$\eta \frac{p(t)}{v(t)} - mgsin\theta - C_r mgcos\theta - 29.17\% \left(\frac{1}{2} C_d A \rho v^2(t) + \frac{7}{8} C_w \pi r^2 v^2(t) \right) = \left(m + \frac{I_f + I_r}{r^2} \right) \frac{dv(t)}{dt} \tag{20}$$

Equation 14 describes that only one break of the power profile is made at the ith section. In this model, the riders will alternate at the front of the pack and will therefore frequently break their power curve limits. Later on, the analysis will be performed for the team as a whole and the equation 14 will be modified as follows

$$\sum_{i=1}^{n} p_i \frac{S_i}{v_{ci}} + \left(\frac{S_i}{6 * 10 v_{ci}} \right) 2\% W_{max} \tag{21}$$

Therefore, the formula 17 will be modified to

$$\min \sum_{i=1}^{n} \frac{S_i}{k_i p_i + b_i}$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} \frac{S_i}{v_{ci}} \left(p_i + \frac{1}{60} 2\% W_{max} \right) \le W_{max} \\ k_i p_i + b_i \le v_{imax} \quad i = 1, \cdots, n \\ \frac{S_i}{k_i p_i + b_i} \le W_i \left(\frac{1}{P_i - CP_i} - \frac{1}{Pi_{max} - CP_i} \right) + 30 \quad i = 1, \cdots, n \end{cases} \tag{22}$$

According to the modified formula, using the particle swarm algorithm to optimize the power distribution, we can obtain the distribution of the rider's overall power with the position.

This strategy requires collaboration and cooperation between riders. A person in the team is sometimes at the front of the line and sometimes at the end of the line, and his or her power will change somewhat with position. Therefore, we connected the power of the riders in the middle position of each stage with a curve, and the power change graph during the ride is roughly shown in Figure 20.

4 Error and Sensitivity Analysis of Models

The sensitivity and error analysis of models under the influence of weather are analyzed in 3.3, and the sensitivity analysis of models for the power shift itself is also analyzed in 3.4, so this section will not be repeated.

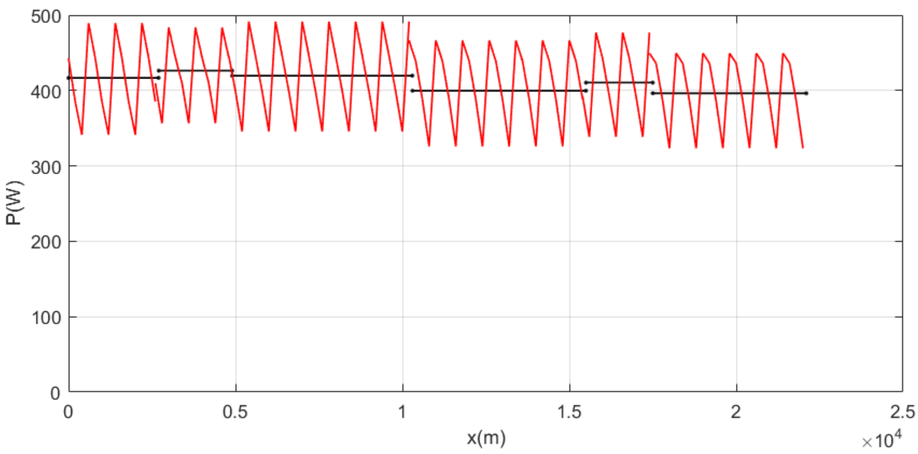

Figure 20: Team and individual power distribution

5 Evaluation and Further Discussion

Advantage

1. **Strong stability** by the sensitivity analysis of wind speed and power deviation from the ideal value of the equation 17 and its derived formula is stable, even if there is a certain deviation in the actual situation, the model can be automatically adjusted to make the optimal choice for the subsequent sections.

2. **Strong scientific** integrated model based on knowledge of kinetics and physiology and combined with mechanistic modeling approach to build and therefore has a well-established scientific background.

3. **We have made reasonable assumptions in the modeling process,** cited references reasonably and adapted them to the situation. We have made reasonable assumptions in the modeling process, cited references reasonably, and adapted them to the situation so that they are highly practical for the actual situation.

4. **The strong usability of this model applies to many scenarios.** Not only can it be used in single-time trials, but also can be migrated to team time trials. In addition to cycling scenarios, it can also be used in track and field and marathon events, as well as in sailing and dragon boat events. High promotion and practical value.

Disadvantage

1. **Some complex factors are ignored** such as the rough consideration of wind speed in this model, which does not take into account the effect of wind direction change on the model. The effect of wind direction change on the model is not considered.

2. **Longer time for a model solution.** This model uses a nonlinear optimization model and combines a particle swarm simulation algorithm to solve it, which has a high number of iterations and a high complexity of the algorithm. The number of iterations is high and the complexity of the algorithm is high, so the solution time is long.

6 Guidance for a Directeur Sportif of a team

Dear Mr. Directeur Sportif:

Our team has developed a model to determine when and where to use A model to determine how much power to use, when and where, is called the integrated model. The integrated model can easily assist you in guiding your riders to use different power to achieve better race results. It is also useful for situations where the rider is not able to achieve the desired power. The integrated model can be used to advise on the power to be used on subsequent rides. The integrated model also has some stability for wind speed.

We would like to give you some advice on how to use the integrated model.

First, we define the power curve for different types of riders and take into account the influence of gender. You can apply your rider to our classification. We classify riders into several categories: all-rounder, sprinter, time trialist, and pursuiter. The time trial specialist is the rider with the highest speed retention, who has the highest critical power, which means he can ride at a higher speed. This means that he can ride for longer periods at a higher speed, which is often the key to winning. Therefore, the above power curve matches the actual situation.

Using a road-based kinetic model, together with consideration of the rider's total energy limitations, we developed a model of the rider's proposed power and corresponding The model is based on the road dynamics and the rider's total energy limitation. The main road sections considered in this model are straight roads, piked roads with little slope, and curved roads. The model considers the rider's total The model also takes into account the total energy of the rider and the effect of short sprints on the rider.

The power curve shows that frequent changes in current power are extremely taxing on the rider's physical energy, so the guideline for individual time trials is "maintain a steady power output for long periods".

The model uses the Tokyo Olympics as an example to derive the relationship between power and speed for different sections of the course. So by simply providing a gpx file of the race route or other route information, we can use the model to obtain the best power combination and determine the ideal power for different sections of the course. Of course, all of this is done to minimize the time and get the best results.

The model also takes into account the influence of weather, mainly the wind direction and speed. Experimentally we know that the wind speed affects the rider by about 10% of the time at most. If the course is a closed-loop, then you only need to consider particularly windy conditions because they cancel each other out. With the model, you can guide the rider to achieve the best results in the natural wind.

In addition, our model can be applied to team competitions if you need it.

Finally, we have proven that our model is universal and stable for different situations. Often, riders will not be able to ride perfectly at the power provided by the model, but the model can be adjusted so that even if the ideal power is not achieved on one section, the optimal combination of power can be achieved on subsequent sections. So you can use the model to help guide your rider with confidence. There is still room for improvement in our model, and we will update it and provide you with the latest model as data is collected. It's all about getting better results for your riders and your team.

Wish you have a nice day!

Yours
Team #2220079

7 Rerenrences

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